Kinematics 1

$v_{avg} = \frac{\Delta x}{\Delta t}$ $a_{avg} = \frac{\Delta v}{\Delta t}$

$v_{inst} = \frac{dx}{dt}$ $a_{inst} = \frac{dv}{dt}$

Core

- $\Delta x = v_0 t + \frac{1}{2}at^2$
- $v_f^2 = v_0^2 + 2a\Delta x$
- $v_f = v_0 + at$

Vector Subtraction invert

$\vec{A}$ or $\vec{B}$, $\vec{A}\vec{B} = -\vec{BA}$

Unit Vectors

$\vec{A} + \vec{B} = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j}$

$\vec{A} - \vec{B} = (A_x-B_x)\hat{i} + (A_y-B_y)\hat{j}$

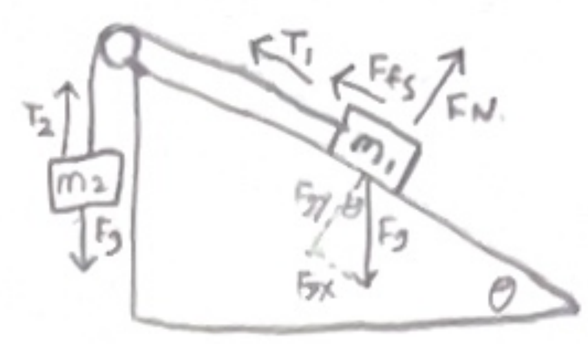
2 Forces

$1N = 1kg \cdot \frac{m}{s^2}$

Newton:

1. $\Sigma F = 0$, then $\vec{a} = 0$
2. $\Sigma F = ma$
3. $F_{AB} = -F_{BA}$

Free Body Diagrams:



$a_{TOT} = \sqrt{a_c^2 + a_T^2} = \sqrt{r^2\omega^4 + r^2\alpha^2}$

$F_{fs} = \mu_s F_N$ * Point of slipping when $F_A = F_{fs}$

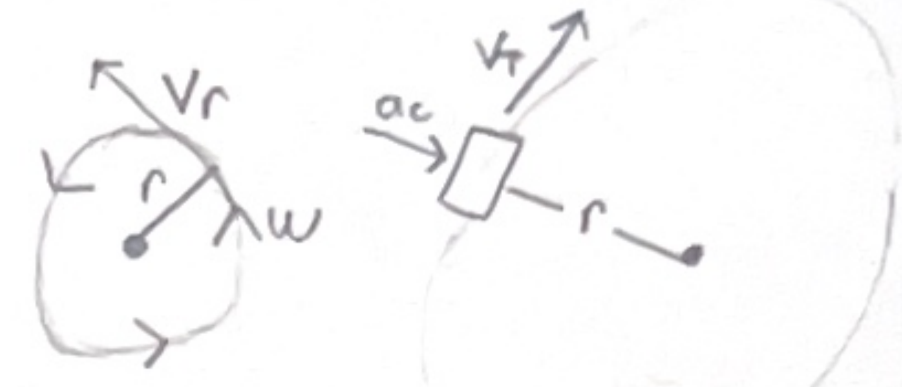
$F_{fk} = \mu_k F_N$ * $\mu_k < \mu_s$

Centripital: $a_c = \omega^2 r$

$F_c = \frac{mv^2}{r}$ $a_c = \frac{v^2}{r}$

$F_c = \frac{m4\pi^2 r}{T^2}$ $V_T = \omega r$

$a_T = r\alpha$



3 Work / Energy

$U_g = -\frac{Gm_1m_2}{r}$

$W_{nc} = \Delta KE + \Delta PE$

$E_i = E_f + \Delta h$ * losses

$W = F\Delta x$

Core:

$E_p = mgh$ $E_k = \frac{1}{2}mv^2$

$E_{ps} = \frac{1}{2}kx^2$

$F_{spring} = -kx$

Power $= \frac{W}{\Delta t}$

$= F \cdot V_{avg}$

watt = J/s

5

4 Momentum

Centre of mass

$x_{cm} = \frac{\Sigma m_i x_i}{m_{total}}$

$y_{cm} = \frac{\Sigma m_i y_i}{m_{total}}$

- $P = mv$

Impulse

- ΔP
- $F\Delta t$

Elastic Collision

1. $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$
2. $KE_i = KE_f$

Inelastic Collision

1. $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$
2. $KE_i \neq KE_f$

perfectly inelastic: Stick together

$v_1' = \frac{(m_1 - m_2)}{(m_1+m_2)} \cdot v_1$

$v_2' = \frac{(2m_1)}{(m_1+m_2)} \cdot v_1$

5 Angular

$x \Rightarrow \theta$
$v \Rightarrow \omega$
$a \Rightarrow \alpha$

Angular Velocity

$\omega = \frac{v}{r}$ $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$

$\omega_{inst} = \frac{d\theta}{dt}$

Rotational kinematics

- $\omega = \omega_0 + \alpha t$
- $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$
- $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Angular acceleration

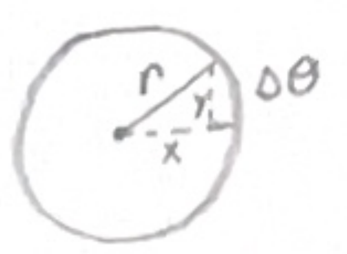
$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$

$\alpha_{inst} = \frac{d\omega}{dt}$

Tangential

$V_T = \omega r$ $\cdot T = \frac{2\pi}{\omega}$

$a_T = \alpha r$



- $x = r\cos\theta$
- $y = r\sin\theta$

Direction of angular velocity

RHR: grab onto the pole and curl your fingers in direction of rotation. Thumb equals $\vec{\omega}$

Direction of angular acceleration

points in the direction in the change($\Delta\omega$) in velocity

Rotational Kinetic Energy

$KE_r = \frac{1}{2}I\omega^2$

Moment of Inertia

$I = \Sigma m_i r_i^2$

Point-mass

$I = mR^2$

Torque

$\tau = I\alpha$

Parallel axis thereom

$I = I_{cm} + Md^2$

* if axis is parallel

$I_{sph} = \frac{2}{5}MR^2 + Md^2 = \frac{2}{5}MR^2$

Angular Momentum

$L = I\omega$

if no external torques

$\Sigma\tau = 0$, momentum conserved

$I_i\omega_i = I_f\omega_f$

Counter $\rightarrow$ (+)

Clockwise $\rightarrow$ (-)

Vertical Spring

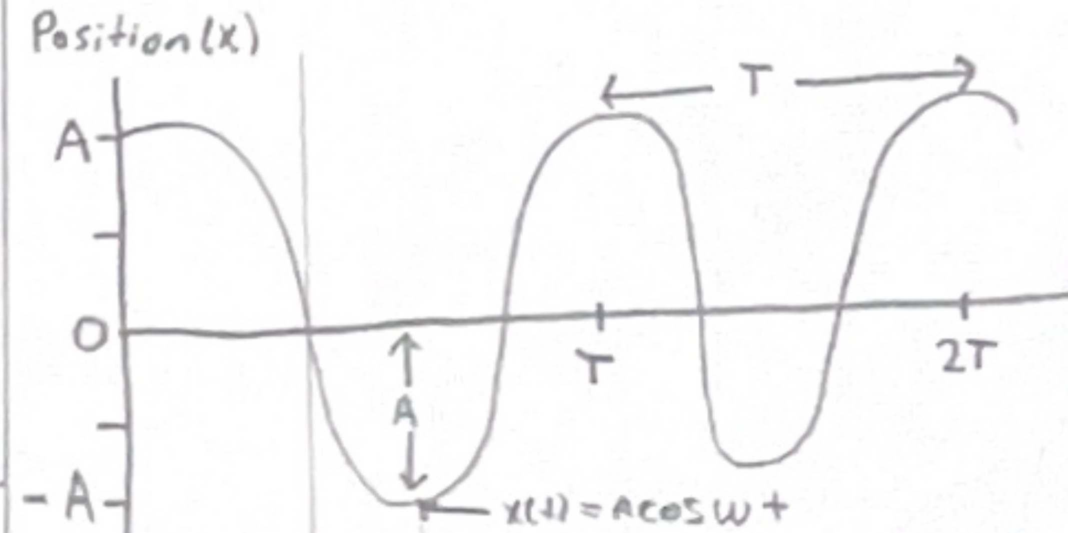
equilibrium position ΔL is

$\Delta L = \frac{mg}{k}$

6 Oscillations / Waves

* The speed is 0 when $x = \pm A$

Simple Harmonic motion

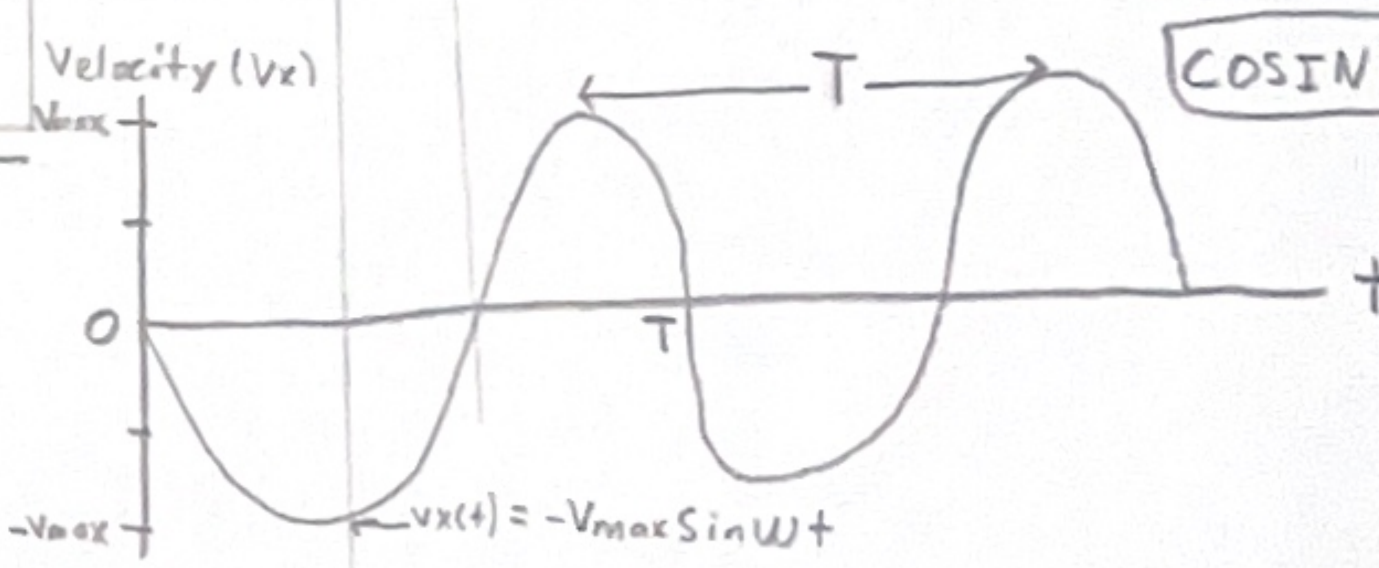


* The speed is max when $x = 0$

SINE

$x(t) = A\cos(\omega t + \phi_0) = A\cos\left(\frac{2\pi t}{T} + \phi_0\right)$

COSINE

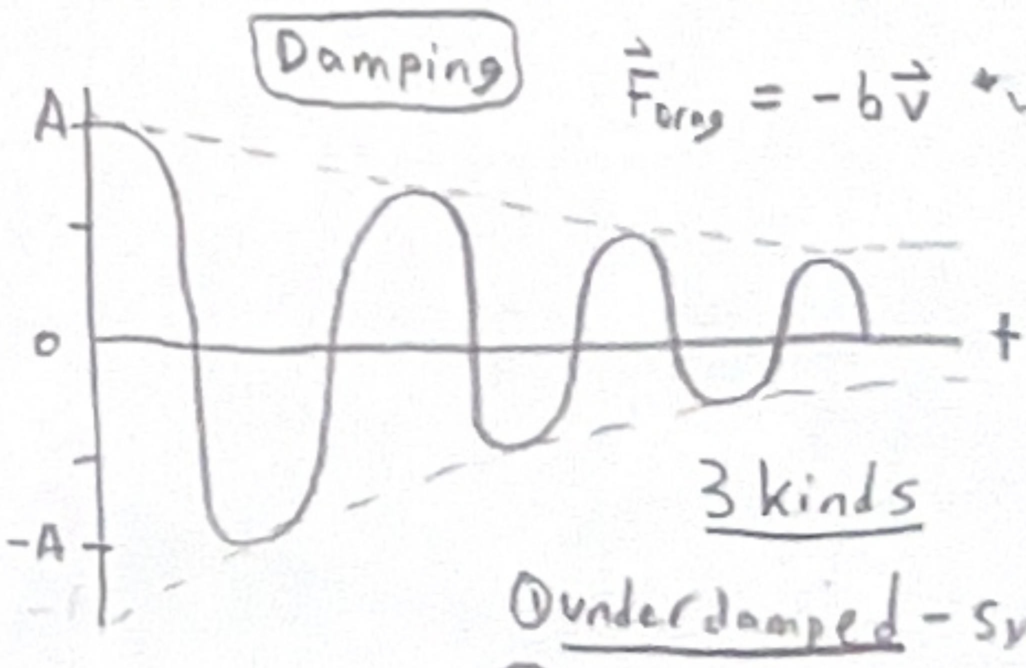


$V_x(t) = -V_{MAX}\sin(\omega t + \phi_0)$, $V_{MAX} = \omega A$

$a_x(t) = -\omega^2 x(t) = -\omega^2 A\cos(\omega t + \phi_0)$

Damping

$\vec{F}_{Drag} = -b\vec{v}$ * where b is damping constant



3 kinds

1. Underdamped - System oscillates
2. Critical damping - System reaches equilibrium most quickly
3. Overdamped - hindered by friction from reaching equilibrium

$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi)$

- $\omega = \sqrt{\frac{k}{m}}$
- $f = \frac{\omega}{2\pi}$
- $V_{MAX} = \sqrt{\frac{k}{m}} \cdot A$
- $T = 2\pi\sqrt{\frac{m}{k}}$

Waves

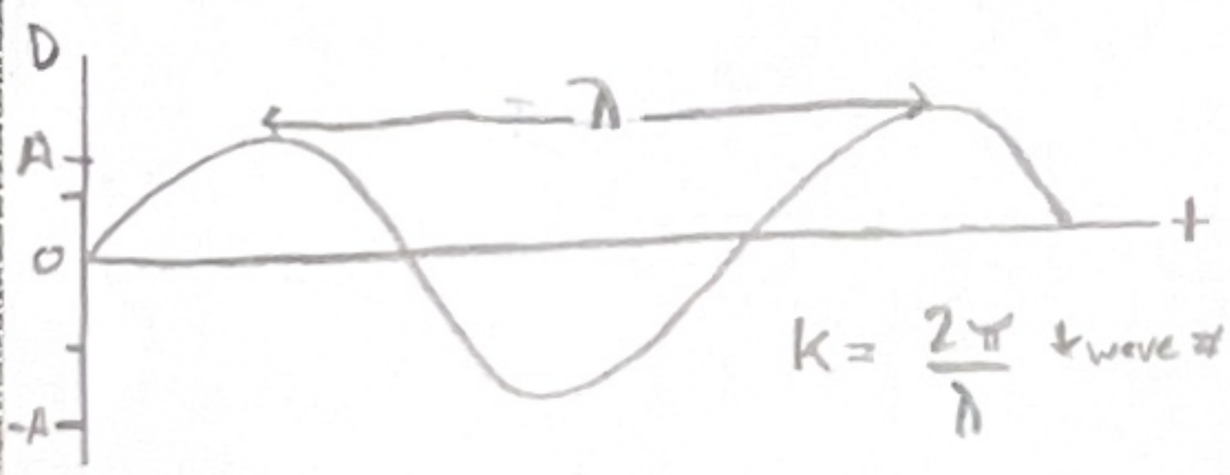
Longitudal $\Rightarrow$ parallel to wave

Transverse $\Rightarrow$ Perpendicular to wave

Wave speed

v = distance / time

$= \frac{\lambda}{T} = \lambda \cdot f$

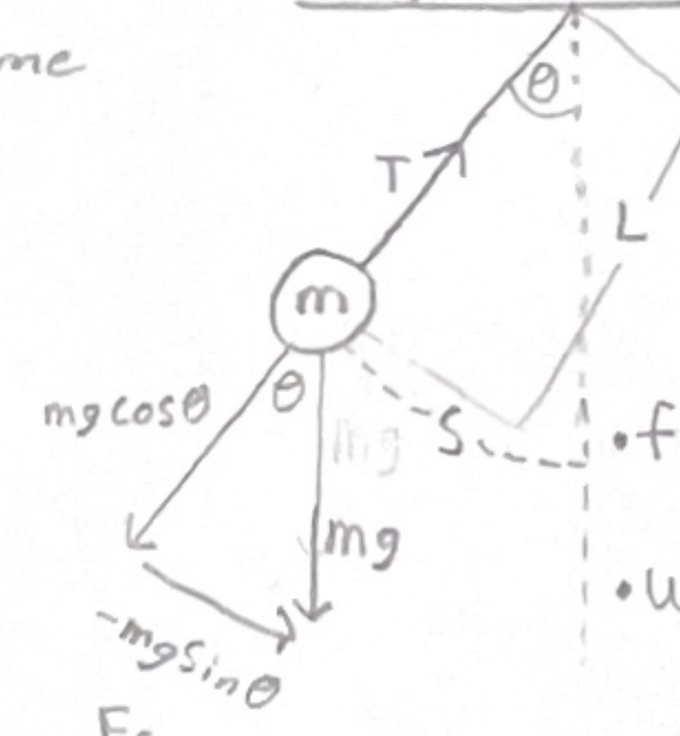


$k = \frac{2\pi}{\lambda}$ #wave #

$D(x,t) = A\sin(kx - \omega t + \phi_0)$

$D(x,t) = A_0\sin\left(\frac{2\pi}{T} \pm \frac{2\pi x}{\lambda}\right)$

Simple Pendulum $\cdot T = 2\pi\sqrt{\frac{L}{g}}$



* For small angles

$F_r = -mg\theta$

$F_r = -kx$

- $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$
- $\omega = \sqrt{\frac{g}{L}}$ $\cdot T = \frac{t}{n}$

Torsional Oscillator

$\omega = \sqrt{\frac{k}{I}}$ * where k equals torsional constant

Driven Oscillations

$x(t) = A\cos(\omega_d t + \phi)$

Natural frequency

$\omega_0 = \sqrt{\frac{k}{m}}$

$A(\omega) = \frac{F_0}{m\sqrt{(\omega_d^2 - \omega_0^2) + b^2\omega_d^2/m^2}}$

- $k = \frac{f}{\Delta y}$

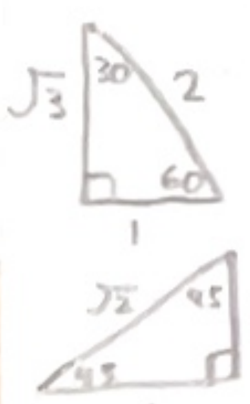
* Δy = Amplitude

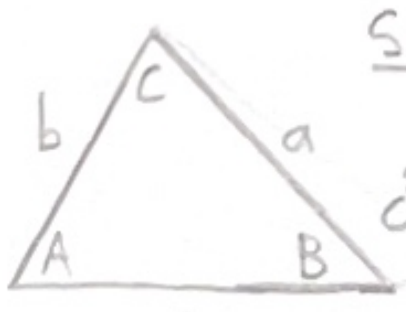
Resonance

$\frac{1}{2}k(A')^2 = \frac{1}{2}k(A_0)^2 + W_{nc}$

$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(V_{max})^2$

Math





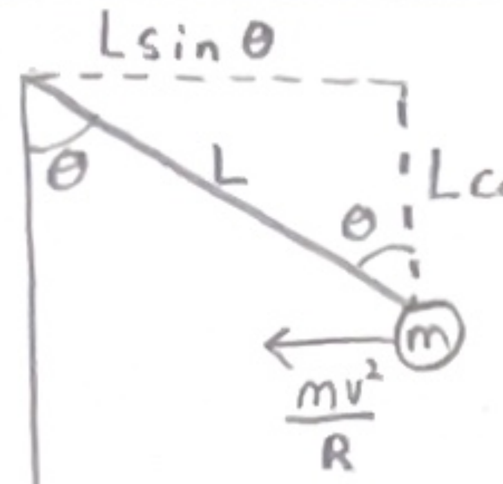
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- $C = 2\pi r$
- $A = \pi r^2$
- $\sin^2 + \cos^2 = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- π radians = 180°
- $A(sphere) = 4\pi r^2$
- $V(sphere) = \frac{4}{3}\pi r^3$

2 A 555kg ball hangs from the top of a vertical pole by a 2.17m long string, the ball is struck causing it to revolve around the pole at a speed of 4.85m/s in a horizontal circle with the string remaining taut. Calculate the angle, between 0° and 90°, that the string makes with the pole.



SOLUTION

$T_r = T\sin\theta$, $T_r = \frac{mv^2}{R}$, $R = L\sin\theta$

$T_{ver} = T\cos\theta$, $T\cos\theta = mg$

$\therefore T\sin\theta = \frac{mv^2}{L\sin\theta}$, $T\sin^2\theta = \frac{mv^2}{L}$

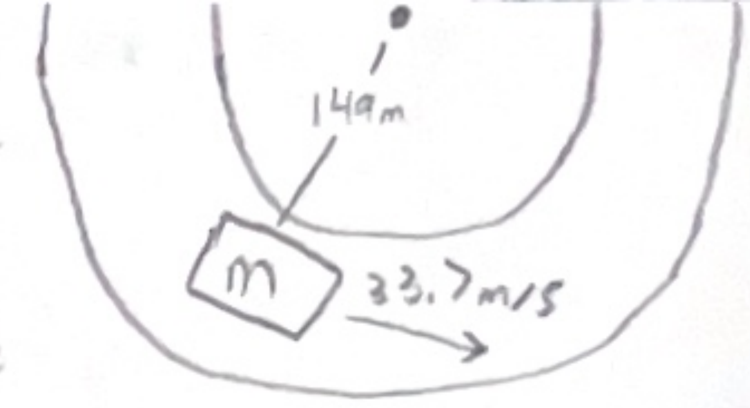
$\frac{T\sin^2\theta}{\cos\theta} = \frac{\left(\frac{mv^2}{L}\right)}{mg}$, $\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{Lg}$

$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v^2}{Lg}$

$Lg\cos^2\theta + v^2\cos\theta - Lg = 0$ QUADRATIC

$\therefore \theta = 53.8°$

2 A 1160kg racecar is driving around a circular track of radius 149m. In the instant shown, the vehicle has a forward speed of 33.7m/s and is slowing at a rate of $5.55 m/s^2$. What is the magnitude of the net force acting on the vehicle at this time and the forces direction relative to velocity.



SOLUTION: $F_T = ma_T$, $(1160)(5.55) = 6440N$

$F_c = ma_c = \frac{mv^2}{r}$, $\frac{(1160)(33.7)^2}{149} = 8840N$

$F_{net} = \sqrt{F_T^2 + F_c^2} = \sqrt{(6440)^2 + (8840)^2} = 10900N$

$\theta = \tan^{-1}\left(\frac{F_T}{F_c}\right)$

3 Given:

$x = 2m$
$k = 50N/m$
$d = 3m$
$h = 5m$
$m = 3kg$
$\mu k = 0.25$

Find: $V_f = ?$

SOLUTION

Before:	After:
$PE_g + PE_s$	$KE + W_{nc}$

$mgh + \frac{1}{2}kx^2 = \frac{1}{2}mvf^2 + F_f \cdot d$

$(3)(9.81)(5) + \frac{1}{2}(50)(2)^2 = \frac{1}{2}(3)vf^2 + (3)(9.81)(0.25)(3)$

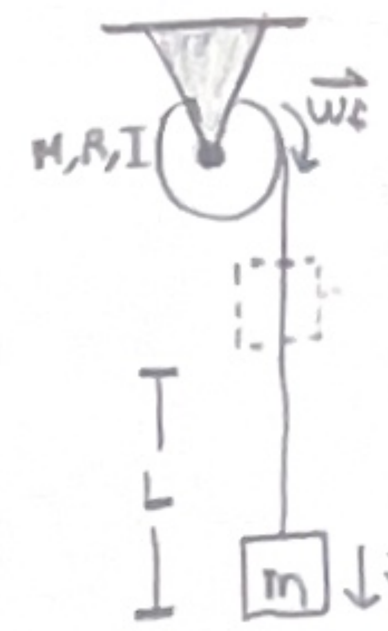
$\therefore \vec{vf} = 12.25 m/s$

5 Given!

$L = 4m$
$R = 0.5m$
$M = 0.5kg$
$m = 5kg$
$I_{cm} = \frac{1}{2}MR^2$

Find: $\vec{vf} = ?$

*note $vf_{cm} = vf_{block}$



Before	After
PE_g	$KE_r + KE_T$

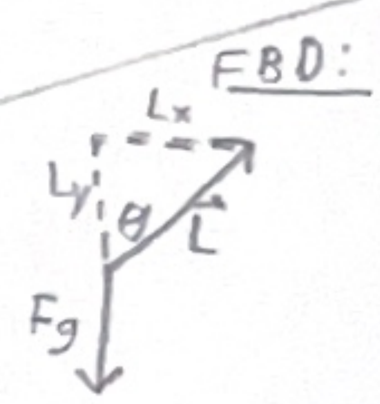
$mgL = \frac{1}{2}I\omega_f^2 + \frac{1}{2}mvf^2$

$mgL = \frac{1}{2}\left(\frac{1}{2}\right)MR^2\left(\frac{vf}{R}\right)^2 + \frac{1}{2}mvf^2$

$\therefore Vf = 8.65 m/s$

$a_c = \frac{4\pi^2 R}{T}$

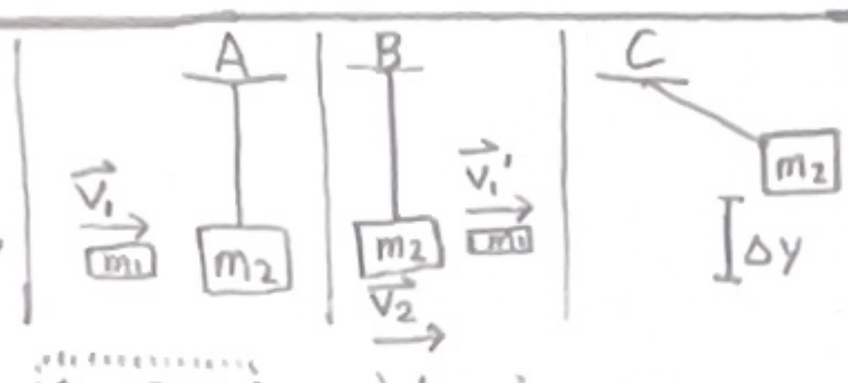
FBD:



4 Given:

$m_1 = 0.007$
$m_2 = 1.5$
$V_1' = 200$
$\Delta y = 0.12$

Find: $V_1 = ?$



SOLUTION:

Before	After
KE	PE

$\frac{1}{2}mv_2^2 = mg\Delta y$

$\therefore V_2' = 1.53 m/s$

$m_1V_1 + \cancel{m_2V_2} = m_1V_1' + m_2V_2'$

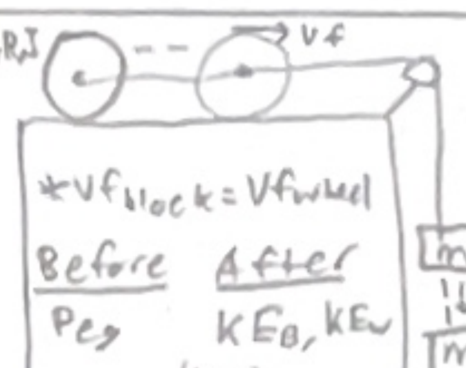
$(0.007)(V_1) = (0.007)(200) + (1.5)(1.53)$

$\therefore V_1 = 528 m/s$

5 Given:

$m = 4$
$M = 8$
$R = 0.5$
$I = 2$
$h = 2$

Find: $Vf = ?$



*$vf_{block} = Vf_{wheel}$

Before	After
Pe_g	KE_B, KE_W $(KE_r)_W$

*static ff is conservative

*Vertical Tension greater for pulley with mass

SOLUTION.

$mgh = \frac{1}{2}mvf^2 + \frac{1}{2}Mvf^2 + \frac{1}{2}I\omega^2$

$(4)(9.81)(2) = \frac{1}{2}(4)vf^2 + \frac{1}{2}(8)vf^2 + \frac{1}{2}(2)\left(\frac{vf}{0.5}\right)^2$

$\therefore Vf = 2.8 m/s$

2 Given:

$m = 8830kg$
$R = 9.11mi$
$T = 0.121hrs$

Find: Lift force = ?

• $\Sigma F_y = L_y + F_g = 0$
• $\Sigma F_x = L_x = ma_c$
• $L_x = \frac{4\pi^2 R}{T}$
• $L_y = mg$

$\vec{L} = \sqrt{L_x^2 + L_y^2} = \sqrt{\left(\frac{4\pi^2 R}{T}\right)^2 + (mg)^2}$

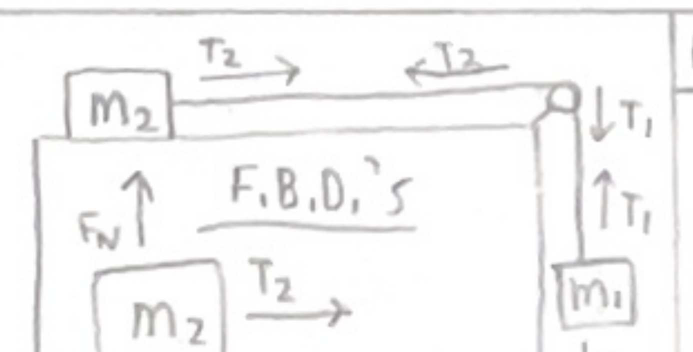
$\therefore \vec{L} = 90.7kn$

$\theta = \tan^{-1}\left(\frac{L_x}{L_y}\right)$

5 Given:

$m_1 = 4$
$m_2 = 3$
$I = 0.5$
$R = 0.3$

Find $\vec{a} = ?$



F.B.D.'s

F_N, m_2, T_2, mg

$\Sigma F_x = T_2 = m_2a$

T_1, m_1, m_1g

$\Sigma F_y = T_1 - m_1g = m_1a$

*note $a_1 = -a_2$

$\alpha = \frac{a}{R^2}$ *For this only

$\Sigma \Upsilon = T_1 - T_2 = I\alpha$

$m_1g - m_1a - m_2a = \frac{Ia}{R^2}$

$(4)(9.81) - (4)(a) - (3)(a) = \frac{(0.5)(a)}{0.3^2}$

$\therefore R = 3.12 m/s$

6 A simple pendulum with a period $T = 1$ second is attached to the ceiling of an elevator which is initially at rest. Calculate the period of oscillation when the elevator begins to rise upwards with $\vec{a} = 0.6 m/s^2$

SOLUTION: $T = 2\pi\sqrt{\frac{L}{g}}$, Find "L"

$T_{up} = 2\pi\sqrt{\frac{L}{g+0.5}} = 0.975s$

6 Given:

$T = 0.8s$
$A = 10cm$
at $t=0$, it is 5cm to the left of equilibrium moving left

Find: Position and direction of motion at $t = 2s$

SOLUTION

$X_o = A\cos(\phi_o)$

$\therefore \phi = \cos^{-1}\left(\frac{x_o}{A}\right) = \cos^{-1}\left(\frac{-5}{10}\right) = \frac{2\pi}{3}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.85 rad/s$

at time $t = 2$

$x(t) = A\cos(\omega t + \phi t)$
$= 10\cos\left(7.85 \cdot 2 + \frac{2\pi}{3}\right)$
$= 5cm$, so the object is 5cm to the right of equilibrium, But which way is it going

SOLUTION $\vec{V}$:

$V(t) = -\omega A\sin(\omega t + \phi_o) = +68$

Positive, so moving to the right

6 Given:

$m = 500kg$
$A = 20cm$
$T = 0.8s$

Find: @ what position is the blocks speed 1m/s.

Find! what is the spring constant

SOLUTION:

$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$\omega = \frac{2\pi}{0.8} = 7.85$

$X = \sqrt{A^2 - \frac{mv^2}{k}} = \sqrt{A^2 - \left(\frac{v}{\omega}\right)^2}$

$x = \sqrt{(0.2)^2 - \left(\frac{1}{7.85}\right)^2} = \pm 15cm$

*block has this speed on either side of equilibrium

SOLUTION! $T = 2\pi\sqrt{\frac{m}{k}}$ *solve for k

6 "Small Angle"

Given:

$m = 300g$
$L = 30cm$
$V = 0.25m/s$ at lowest point

Find: Max angle the pendulum reaches

SOLUTION:

$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.81}{0.30}} = 5.72 rad/s$

at lowest point $V_{MAX} = \omega A$

$A = \frac{V_{MAX}}{\omega} = \frac{0.25}{5.72} = 0.0437M$

$\theta_{max} = \frac{0.0437}{0.30} = 0.146 rad = 8.3°$

Less than 10° so reasonable

6 Swinging Rod Mass not [illegible]

Given:

$L = 0.90m$

Find Frequency and Period

SOLUTION, $I = \frac{1}{3}ML^2$

$f = \frac{1}{2\pi}\sqrt{\frac{Mg\ell}{I}}$

$= \frac{1}{2\pi}\sqrt{\frac{Mg(L/2)}{\frac{1}{3}ML^2}}$

$= \frac{1}{2\pi}\sqrt{\frac{3g}{2L}} = 0.64 Hz$, $T = \frac{1}{f}$

$\therefore T = 1.6s$

6 "Wave"

A sinusoidal wave with amplitude 1cm and frequency 100Hz travels at 200m/s in the positive x direction. At $t=0$, the point $x = 1m$ is on a crest of the wave

Find

• A
• V
• λ
• k
• f
• ω
• T
• φ and

Write displacement equation

What we know from Above

• $A = 1cm$
• $V = 200m/s$
• $f = 100Hz$

SOLUTION:

$\lambda = \frac{v}{f} = 2m$

$k = \frac{2\pi}{\lambda} = 3.14 rad/m$

$\omega = 2\pi f = 628 rad/s$

$T = \frac{1}{f} = 0.01s$

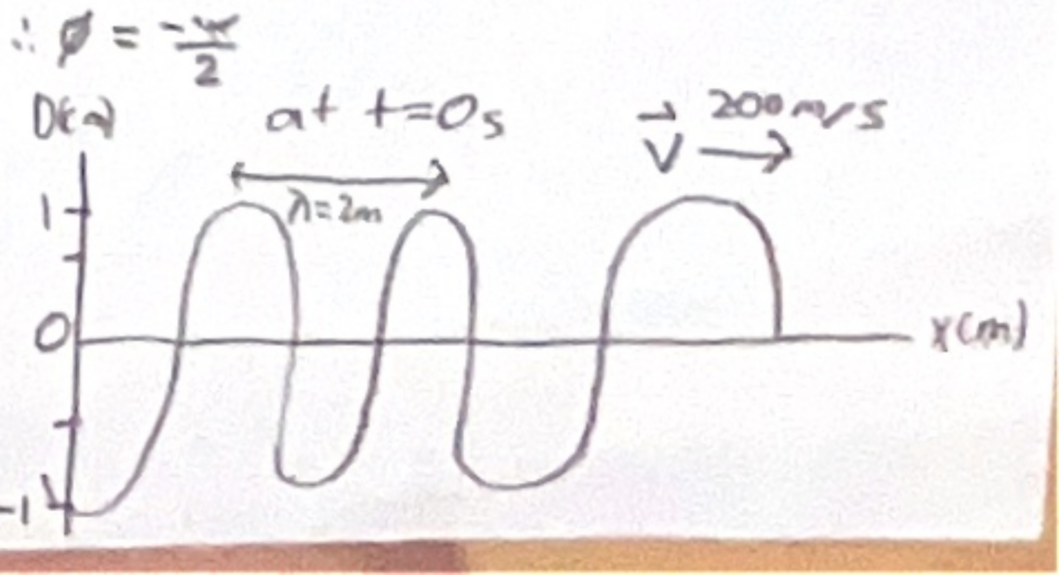
$D(x_0, t_0) = A = A\sin[k(1m) + \phi_o]$

$k(1) + \phi = \frac{\pi}{2}$

$\therefore \phi = \frac{-\pi}{2}$

Wave Equation

$D(x,t) = (1cm)\sin\left[(3.14)x - (628)t - \frac{\pi}{2}\right]$



6 "Damped"

Given:

$m = 500g$
$L = 60cm$

amplitude is observed to decay to half its initial value after 35 oscillations

Find

The time constant "and"

At what time will the energy have decayed to half its initial value

After 35 oscillations $X_{MAX} = \frac{1}{2}A$

SOLUTION:

at t_0, $X_{MAX} = A$

$T = 2\pi\sqrt{\frac{0.60}{9.81}} = 1.55s$

So at 54.25, 35 oscillations have occurred

$X_{max}(t) = Ae^{-bt/2m} = Ae^{-t/2\Upsilon}$

$\frac{1}{2}A = Ae^{(-54.2)/2\Upsilon}$

$\ln\left(\frac{1}{2}\right) = \frac{-54.2}{2\Upsilon}\ln e$

$\therefore \Upsilon = 39$ seconds

SOLUTION:

*note $b = \frac{m}{\Upsilon} = 0.013 kg/s$

$E(t) = E_o e^{-t/\Upsilon}$

*E_o cancels

$\frac{1}{2} = e^{-t_{1/2}/\Upsilon}$

$\ln\left(\frac{1}{2}\right) = \frac{-t_{1/2}}{\Upsilon}\ln e$

$\ln\left(\frac{1}{2}\right) = -\frac{t_{1/2}}{\Upsilon}$

$t_{1/2} = 0.693\Upsilon$

$t_{1/2} = (0.693 \cdot 6)(39s)$

$= 27s$ *oscillator loses E faster than A

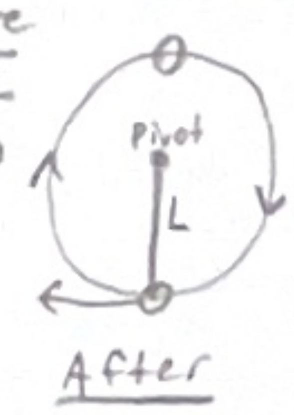
6 "Driven"

Given

Speed at lowest point is 5m/s

if the pendulum gets pushed at the bottom of the arc

Find frequency



Before

$y_i = 2L$
$v_i = 0$

After

$y_f = 0$
$V_f = 5$

$\frac{1}{2}vf^2 = g(2L)$
$= 2gL$

$L = \frac{vf^2}{4g}$

$= 0.638m$

SOLUTION: $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

$= 0.62Hz$

$\frac{1}{2}mvf^2 + \cancel{mgyf} = \frac{1}{2}mv_i^2 + mgy_i$